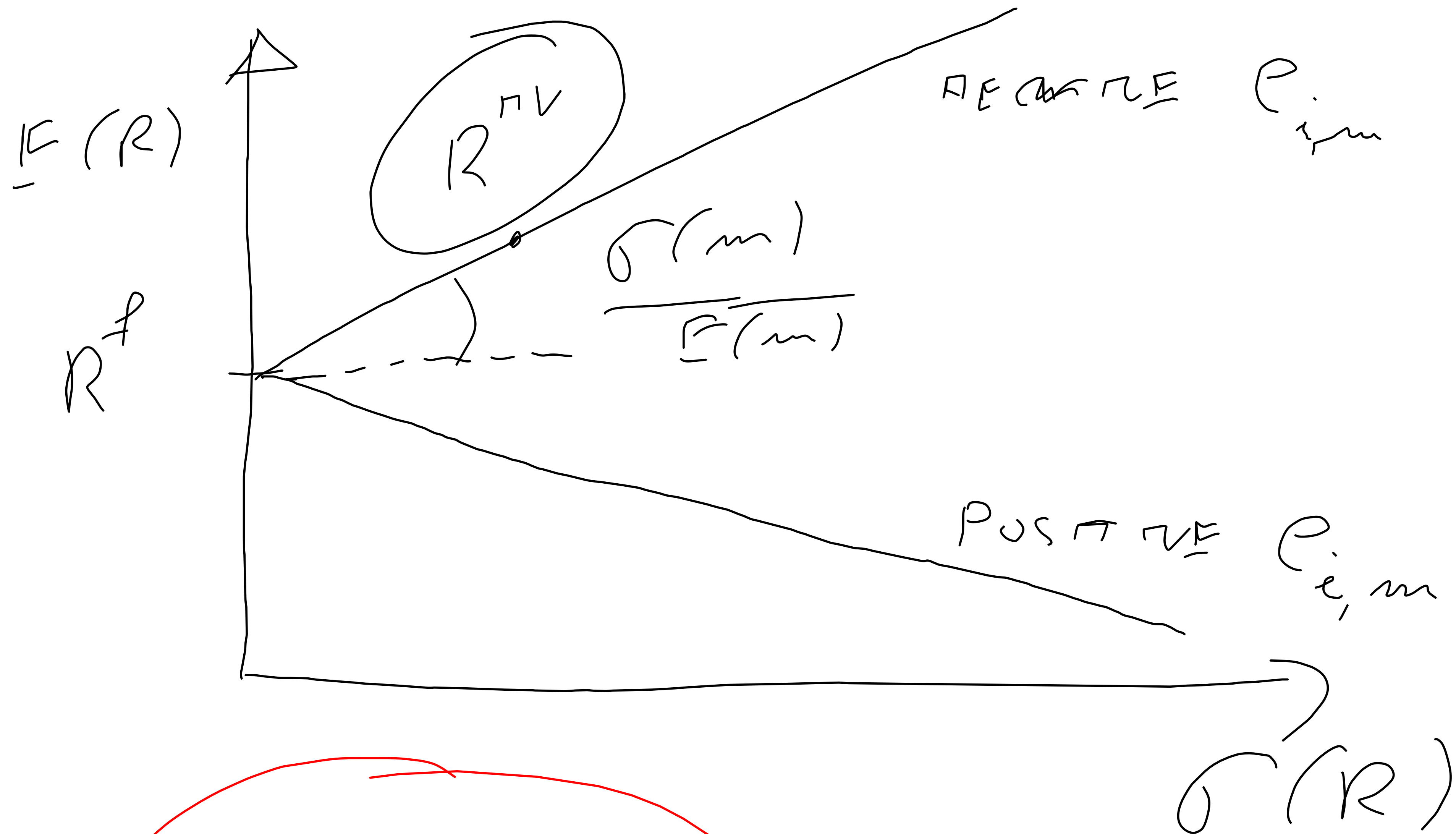
$$\left| \frac{E(R^i) - R^f}{\sigma(R^i)} \right| \leq \frac{\sigma(m)}{E(m)}$$

$$|\rho_{i,m}| \leq 1$$

$$|\rho_{i,m}| = 1$$

MAX SHARPE RATIO $= \dfrac{\sigma(m)}{E(m)}$



IF $|e| < 1$ $\Rightarrow$

$m = a + b\,R^{MV}$

$R^{MV} = e + g\,m$

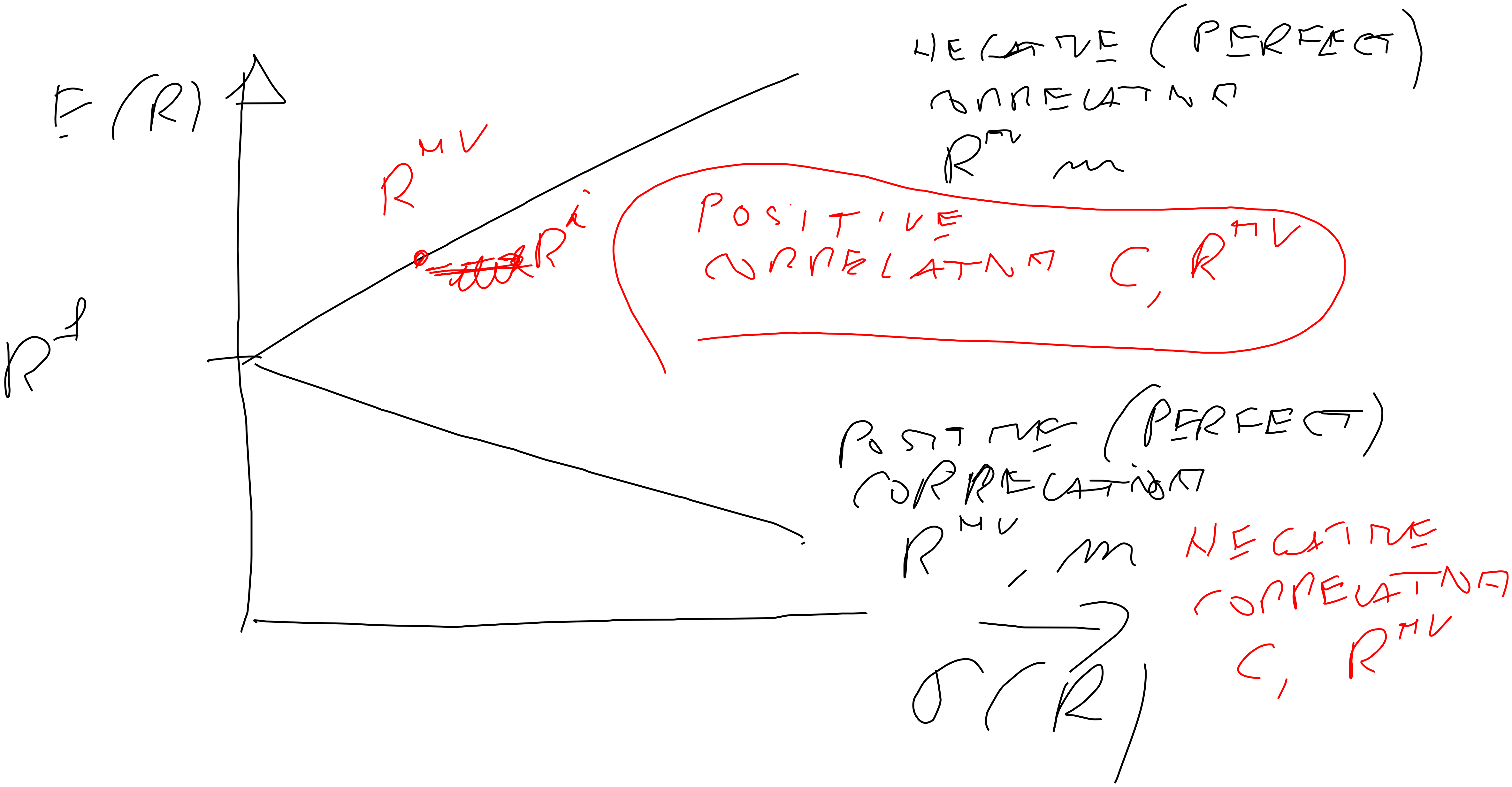
$$\rho = \frac{cov(m, R^{MV})}{\sigma(m)\,\sigma(R^{MV})}$$

$$m = a + b R^{MV}$$

$$cov(m, R^{MV}) = cov\left((a + b R^{MV}), R^{MV}\right) = b\, VAR(R^{MV})$$

$$\sigma(m) = \sigma(a + b R^{MV}) = |b|\, \sigma(R^{MV})$$

$$\rho = \frac{b\, VAR(R^{MV})}{|b|\, \sigma(R^{MV})\, \sigma(R^{MV})} = \frac{b}{|b|} \cancel{\frac{VAR(R^{MV})}{VAR(R^{MV})}} \begin{cases} +1 \\ -1 \end{cases}$$



E(R)
R^f
R^MV
R^i
NEGATIVE (PERFECT) CORRELATION
R^MV, m
POSITIVE CORRELATION C, R^MV
POSITIVE (PERFECT) CORRELATION
R^MV, m
NEGATIVE CORRELATION
C, R^MV
σ(R)

$$\boxed{m = a + b\, R^{MV}}$$

$$1 = E\left[m R^i\right] \Rightarrow \simeq \text{CAPM}$$

WE KNOW THAT

$\text{COV}(R^i, m)$

EXPECTED RETURN-BETA REPRESENTATION

$$E(R^i) = R^f + \beta_{i,m} \lambda_m$$

$$\lambda_m = \left(- \frac{\text{VAR}(m)}{E(m)} \right)$$

$$m = a + b R^{MV}$$

DERIVE a, b BY

PRICING TWO ASSETS

$$\begin{cases} 1 = E\left[m R^{MV}\right] = E\left[\left(a + b R^{MV}\right) R^{MV}\right] = a E\left(R^{MV}\right) + b E\left[\left(R^{MV}\right)^2\right] \\ 1 = E\left[m R^{f}\right] = E\left[\left(a + b R^{MV}\right) R^{f}\right] = a R^{f} + b R^{f} E\left(R^{MV}\right) \end{cases}$$

2 EQUATIONS IN THE UNKNOWNS a, b

3

$$b = -\frac{1}{R^f}\,\frac{\left(E(R^{MV}) - R^f\right)}{VAR(R^{MV})}$$

EXPECTED RETURN - BETA REPRESENTATION

1

$$E(R^i) = R^f + \beta_{i,m}\,\lambda_m$$

2 $m = a + b R^{MV}$

$$= R^f + \frac{COV(R^i, m)}{VAR(m)}\left(-\frac{VAR(m)}{E(m)}\right)$$

$$= R^f + \frac{COV(R^i, a + b R^{MV})}{VAR(a + b R^{MV})}\left(-\frac{VAR(a + b R^{MV})}{1/R^f}\right)$$

$$E(R^i) = R^f + \frac{b \, cov(R^i, R^{MV})}{b^2 \cancel{VAR(R^{MV})}} \left(-R^f \cancel{b^2 VAR(R^{MV})} \right)$$

$$E(R^i) = R^f - R^f b \, cov(R^i, R^{MV})$$

$$= R^f - R^f \left(-\frac{1}{R^f} \frac{(E(R^{MV}) - R^f)}{VAR(R^{MV})} \right) cov(R^i R^{MV})$$

RETURN ON THE FRONTIER

$$= R^f + \beta_{i R^{MV}} \left(E(R^{MV}) - R^f \right)$$

CAPM

$$\max \left| \frac{\underline{E}(R^{MV}) - R^f}{\sigma(R^{MV})} \right| = \frac{\sigma(m)}{\underline{E}(m)}$$

$\leq$ (If return is not on the frontier)

COMPUTE MAX SHARPE RATIO

UNDER THE MODEL

$$m = e^{-\delta - \gamma(\Delta c)}$$

$E(m) =$ PRICE OF A BOND

$$\Delta c = \ln(c_{t+1}) - \ln(c_t)$$

$$\Delta c \sim N\left(E(\Delta c), \sigma^2(\Delta c)\right)$$

$$\frac{\sigma(m)}{E(m)}$$

$$E(m) = E\left[e^{-\delta - \gamma(\Delta c)}\right] \quad \Delta c \sim N(\quad)$$

$$= e^{-\delta - \gamma E(\Delta c) + \gamma^2/2\, \sigma^2(\Delta c)}$$

$$\sigma^2(m) = E(m^2) - E(m)^2 = \text{VARIANCE OF } m$$

$$E(m^2) = E\left[\left(e^{-\delta - \gamma(\tilde{\Delta c})}\right)^2\right] = E\left[e^{-2\delta - 2\gamma(\tilde{\Delta c})}\right]$$

$$E(m^2) = E\left[e^{-2\delta - 2\gamma(\Delta c)}\right] =$$

$$= e^{-2\delta - 2\gamma E(\Delta c) + \frac{4}{2}\gamma^2 \sigma^2(\Delta c)}$$

$$E(m)^2 = \left(e^{-\delta - \gamma E(\Delta c) + \frac{\gamma^2}{2}\sigma^2(\Delta c)}\right)^2$$

$$= e^{-2\delta - 2\gamma E(\Delta c) + \gamma^2 \sigma^2(\Delta c)}$$

$$\mathrm{var}(m) = \sigma^2(m) = e^{-2\delta - 2\gamma E(\Delta c) + 2\gamma^2\sigma^2(\Delta c)} - e^{-2\delta - 2\gamma E(\Delta c) + \gamma^2\sigma^2(\Delta c)}$$

$$VAR(m) = e^{-2\delta - 2\gamma E(\Delta c) + \gamma^2 \sigma^2(\Delta c)} \cdot$$

$$\left(e^{\gamma^2 \sigma^2} - 1\right) =$$

$$= E(m)^2 \left(e^{\gamma^2\sigma^2} - 1\right)$$

$$\sqrt{VAR(m)} = \sigma(m) = E(m)\sqrt{e^{\gamma^2\sigma^2} - 1}$$

MAX SHARPE RATIO $= \dfrac{\sigma(m)}{E(m)} = \sqrt{e^{\gamma^2\sigma^2} - 1}$

$$\frac{\sigma(m)}{E(m)} = \sqrt{e^{\gamma^2\sigma^2} - 1}$$

$$\simeq \sqrt{\text{TAYLOR EXPAND AROUND } \gamma^2\sigma^2 = 1}$$

$$= \sqrt{\gamma^2 \sigma^2(\Delta c)} = \gamma\,\sigma(\Delta c)$$

RISK AVERSION

$$\frac{\sigma(m)}{E(m)} \simeq \gamma\ \sigma(\Delta c)$$

STD OF CHANGES IN LOG CONSUMPTION

$$m = e^{-\delta - \gamma(\Delta c)}$$

US MARKET

50 YEARS

1950 → 2000

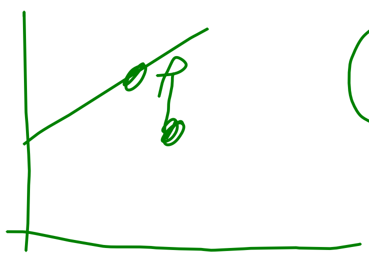
9% 1%

$$\frac{E(R) - R^f}{\sigma(R)} = 0,5 = \gamma\,\sigma(\Delta c)$$

0,5% 16%

0,01

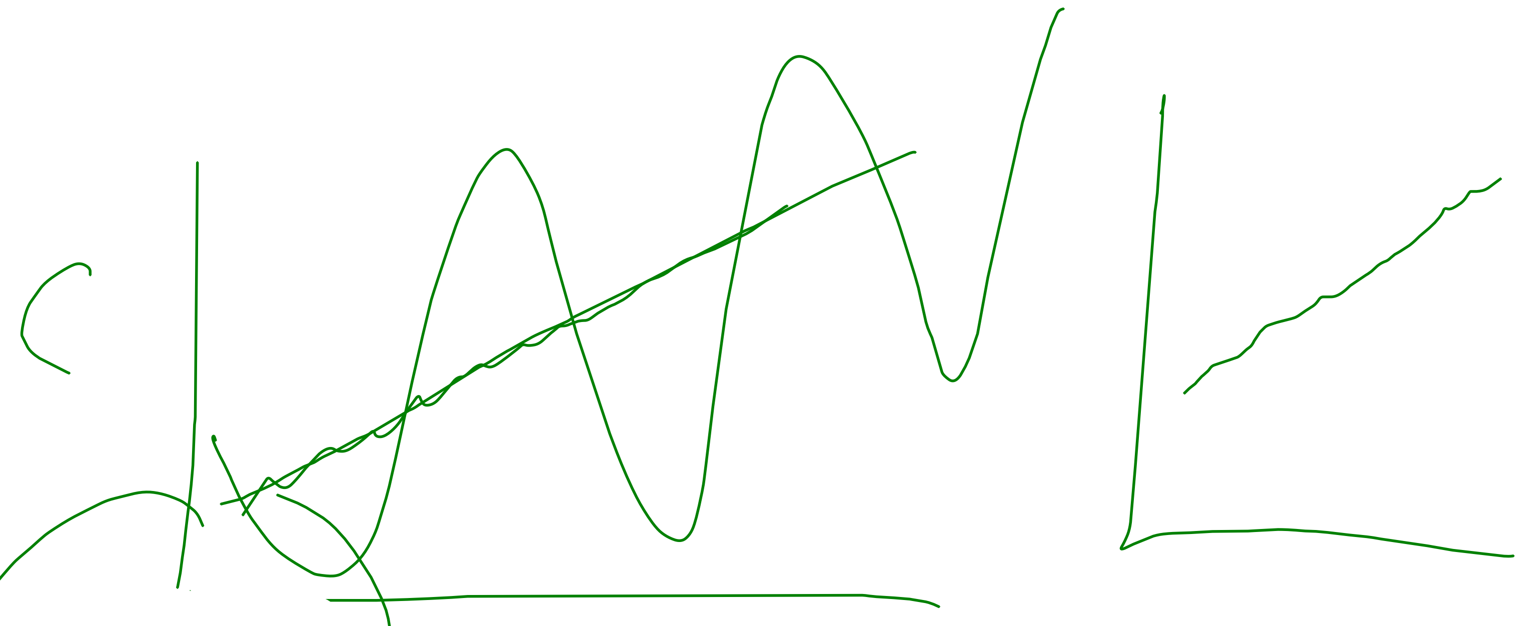
$\gamma = 50$ 250



$\rho < 1$

$0 < \gamma < 1$

$m = e^{-\delta - \gamma \Delta c}$



t

$u(c_t) + E(u(c_{t+1}))$

$u(c_{t+1} - c_t)$

MODEL FOR u THAT MAKES IT VERY VOLATILE

(EXPECTED UTILITY IS NOT ADDITIVE)

L

1 50

$$R^f = \delta + \gamma E(\Delta c) \pm \gamma^2 \sigma^2(\;)$$